HIGHER SECONDARY FIRST YEAR

MATHEMATICS

MODEL QUESTION PAPER

Time Allowed: 2.30 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

SECTION – I

Note: (i) **All** questions are **compulsory.** $20\times 1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If two sets A and B have 17 elements in common, then the number of elements common to the set $A\times B$ and $B\times A$ is

   (1) 2^{17} (2) 17^2 (3) 34 (4) insufficient data

2. If $\mathbb{R}$ is the set of all real numbers and if $f:\mathbb{R}-\{3\}\to\mathbb{R}$ is defined by $f(x)=\dfrac{3+x}{3-x}$ for $x\in\mathbb{R}-\{3\}$, then the range of f is

   (1) $\mathbb{R}$ (2) $\mathbb{R}-\{1\}$ (3) $\mathbb{R}-\{-1\}$ (4) $\mathbb{R}-\{-3\}$

3. If the sum and product of the roots of the equation $2x^2+(a-3)x+3a-5=0$ are equal, then the value of a is

   (1) 1 (2) 2 (3) 0 (4) 4

4. Which one of the following is not true?

   (1) $|\sin x|\le 1$ (2) $|\sec x|<1$

   (3) $|\cos x|\le 1$ (4) $\operatorname{cosec} x\ge 1$ or $\operatorname{cosec} x\le -1$

5. $\cos 1^\circ+\cos 2^\circ+\cos 3^\circ+\cdots+\cos 179^\circ$ is

   (1) 0 (2) 1 (3) -1 (4) 89

6. If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent,then the total number of points of intersection are

   (1) 45 (2) 40 (3) 10! (4) 2^{10}

7. The remainder when 2^{2020} is divided by 15 is

   (1) 4 (2) 8 (3) 1 (4) 2

8. The harmonic mean of two positive numbers whose arithmetic mean and geometric mean are 16, 8 respectively is

   (1) 10 (2) 6 (3) 5 (4) 4

9. In the equation of a straight line $ax+by+c=0$, if a,b,c are in arithmetic progression then the point on the straight line is

   (1) $(1,2)$ (2) $(1,-2)$ (3) $(2,-1)$ (4) $(2,1)$

10. If two straight lines $x+(2k-7)y+3=0$ and $3kx+9y-5=0$ are perpendicular to each other then the value of k is

    (1) 3 (2) $\dfrac{1}{3}$ (3) $\dfrac{2}{3}$ (4) $\dfrac{3}{2}$

11. If $|\vec{a}|=13$, $|\vec{b}|=5$ and $\vec{a}\cdot\vec{b}=60^\circ$ then $|\vec{a}\times\vec{b}|$ is

(1) 15 (2)35 (3)45 (4) 25

12. A vector $\overrightarrow{OP}$ makes 60° and 45° with the positive direction of the x and y axes respectively. Then the angle between $\overrightarrow{OP}$ and the z-axis is

(1) 45° (2) 60° (3) 90° (4) 30°

13. A vector perpendicular to both $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$ is,

(1) $2\hat{i}+\hat{j}-\hat{k}$ (2) $2\hat{i}-\hat{j}-\hat{k}$ (3) $3\hat{i}+\hat{j}+2\hat{k}$ (4) $3\hat{i}+\hat{j}-2\hat{k}$

14. $\lim\limits_{x\to 0}\dfrac{\sin|x|}{x}$ is

(1) 1 (2) -1 (3)0 (4) does not exist

15. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=\lfloor x-3\rfloor+|x-4|$, $x\in\mathbb{R}$, then $\lim\limits_{x\to 3^-}f(x)$ is equal to

(1) -2 (2) -1 (3) 0 (4) 1

16. If $f(x)=\begin{cases} x^3, & x<0 \\ 3a+x^2, & x\geq 0 \end{cases}$ is continuous at $x=0$, then a is

(1) -2 (2) -1 (3) 0 (4) 1

17. The derivative of $f(x)=x|x|$ at $x=-3$ is

(1) 6 (2) -6 (3) does not exist (4) 0

18. $\displaystyle\int\frac{dx}{x(x+1)}$ is

(1) $\log\left|\dfrac{x+1}{x}\right|+c$ (2) $\log\left|\dfrac{x}{x+1}\right|+c$ (3) $\log\left|\dfrac{x-1}{x}\right|+c$ (4) $\log\left|\dfrac{x}{x-1}\right|+c$

19. $\int 2^{3x+5}dx$ is

(1)$\dfrac{3(2^{3x+5})}{\log 2}+c$ (2)$\dfrac{2^{3x+5}}{2\log(3x+5)}+c$ (3)$\dfrac{2^{3x+5}}{2\log 3}+c$ (4)$\dfrac{2^{3x+5}}{3\log 2}+c$

20. If X and Y be two events such that $P(X/Y)=\dfrac{1}{2}$, $P(Y/X)=\dfrac{1}{3}$ and $P(X\cap Y)=\dfrac{1}{6}$, then $P(X\cup Y)$ is

(1) $\dfrac{1}{3}$ (2)$\dfrac{2}{5}$ (3)$\dfrac{1}{6}$ (4)$\dfrac{2}{3}$

SECTION – II

Note: (i) Answer any **SEVEN** questions**.** $7\times 2=14$

(ii) Question number **30 is compulsory.**

21. From the graph $y=\cos x$, draw $|y|=\cos x$.

22. If $\dfrac{\log(x)}{y-z}=\dfrac{\log(y)}{z-x}=\dfrac{\log(z)}{x-y}$, then prove that $xyz=1$.

23. Show that $\tan(45^\circ-A)=\dfrac{1-\tan A}{1+\tan A}$

24. How many ways are there to arrange the letters of the word “GARDEN” with vowels in the alphabetical order.

25. Find the sum $1+\dfrac{4}{5}+\dfrac{7}{25}+\dfrac{10}{125}+\cdots$

26. Show that the points whose position vectors are $2\hat{i}+3\hat{j}-5\hat{k}$, $3\hat{i}+\hat{j}-2\hat{k}$ and $6\hat{i}-5\hat{j}+7\hat{k}$ are collinear.

27. Examine the continuity of the function $\frac{x^2-16}{x+4}$

28. Find the derivative of $y=\log_{10} x$ with respect to x.

29. Evaluate: $\int \frac{\sin x}{1+\cos x}dx$

30. If $A=\begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$ and $(A-2I)(A-3I)=O$, find the value of x.

SECTION – III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) Question number **40 is compulsory.**

31. Check the relation $R=\{(1,1),(2,2),(3,3),\ldots,(n,n)\}$ defined on the set $S=\{1,2,3,\ldots,n\}$ for the three basic relations.

32. Prove that $\frac{\cot(180^\circ+\theta)\sin(90^\circ-\theta)\cos(-\theta)}{\sin(270^\circ+\theta)\tan(-\theta)\operatorname{cosec}(360^\circ+\theta)}=\cos^2\theta\cot\theta$.

33. In an examination a student has to answer 5 questions out of 9 questions, in which 2 are compulsory. In how many ways a student can answer the questions?

34. Find the coefficient of x^{15} in $\left(x^2+\frac{1}{x^3}\right)^{10}$.

35. Find the equations of the straight lines, making the y-intercept of 7 and angle between the line and the y-axis is 30°.

36. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}=(x-y)(y-z)(z-x)$.

37. If $\vec{a},\vec{b}$ and $\vec{c}$ are vectors with magnitudes 3,4 and 5 respectively and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

38. Evaluate : $\int x\log x\, dx$.

39. If A and B are mutually exclusive events $P(A)=\frac{3}{8}$ and $P(B)=\frac{1}{8}$, then find

(i) $P(\bar{A})$ (ii) $P(A\cup B)$ (iii) $P(\bar{A}\cap B)$

40. Evaluate : $\lim\limits_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$.

SECTION – IV

Note: Answer **all** the questions. $7\times5=35$

41. (a) If $f,g:\mathbb{R}\to\mathbb{R}$ are defined by $f(x)=|x|+x$ and $g(x)=|x|-x$, find $g\circ f$ and $f\circ g$.

(OR)

(b) Solve the linear inequalities and exhibit the solution set graphically:

$x+y\geq 3, 2x-y\leq 5, -x+2y\leq 3$.

42. (a) If $A+B+C=\pi$, prove that $\cos A+\cos B+\cos C=1+4\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$

(OR)

(b) In a ΔABC, prove that $a\cos A+b\cos B+c\cos C=2a\sin B\sin C$.

43. (a) Prove by the principle of mathematical induction, the sum of the first n non-zero even numbers is n^2+n.

(OR)

(b) The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and the n^{th} hour?

44. (a) Show that $\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}=0$

(OR)

(b) Show that the vectors $\hat{i}-2\hat{j}+3\hat{k}, -2\hat{i}+3\hat{j}-4\hat{k}, -\hat{j}+2\hat{k}$ are coplanar.

45. (a) Describe the interval(s) on which the function $h(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ is continuous.

(OR)

(b) If $\sin y=x\sin(a+y)$, then prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}, a\neq n\pi$.

46. (a) Using the substitution $2x+1=t^2$, show that $\int\frac{6x}{\sqrt{2x+1}}dx = 2(x-1)\sqrt{2x+1}+c$.

(OR)

(b) A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?

47. (a) At a particular moment, a student needs to stop his speedybike to avoid a collision with the barrier ahead at a distance 40 metres away from him. Immediately he slows (retardation) the bike under braking at a rate of 8 metre/second^2. If the bike is moving at a speed of 24m/s,when the brakes are applied, would it stop before collision?

(OR)

(b) Find the separate equations of the pair of straight lines $2x^2-xy-3y^2-6x+19y-20=0$.

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